Homework 4

1. More on cross-sections

   Thus far, we have only made geometrical arguments on what the cross-section should look like. This is largely based on our own physical intuition. We discussed how quantum mechanics makes nuclides fuzzy and how this can increase their effective radii of interaction. Quantum mechanics has another effect on cross-sections that we must take into account, namely that same fuzziness. However this time, the fuzziness is not from the radii of nuclides, but in their positions (or relative position) of the two interacting nuclides. There is some probability of the two nuclides touching or penetrating each others surfaces, where they can then shuffle nucleons around to perform the reaction. This is called the penetration factor. It is given by:

$$P_\ell(E, R_N) = \frac{2\pi\eta k R_N}{\exp(2\pi\eta) - 1} w_\ell(E, R_N) \tag{1}$$

   where $E = \frac{1}{2}\mu v^2$, $\vec{q} = \hbar\vec{k}$, and $w_\ell(E)$ is a polynomial in E of order ℓ. k is called the wavenumber, it is a useful parameterization of the relative momentum q with units of $[L^{-1}]$. Note that the deBroglie wavelength is $\lambda = k^{-1}$. We define the Sommerfeld parameter:

$$\eta = \alpha Z_1 Z_2 \frac{c}{v} = \sqrt{\frac{E_g}{E}} \tag{2}$$

   where E_g is the Gamow energy. This is not to be confused with the "Gamow window". We will discuss that later. Here, α is the fine-structure constant.

   (a) Evaluate the expressions for η and E_g, expressing E_g in terms of α, Z_1, Z_2, μ, $\hbar$ and c.

   (b) Evaluate E_g assuming $\mu = m_u$ and $Z_1 = Z_2 = 1$.

   (c) Neutrons have no charge, therefore $E_g = 0$. Evaluate the penetration factor in the limit that $E_g \to 0$. Assume $\ell = 0$, therefore $w_\ell(E, R_N) = 1$. If you expressed the neutron penetration factor as proportional to the speed v, what power does this proportionality have?

   (d) What power of v does the factor $2\pi\eta k R_N$ have?

   Now we can put together the gross behavior of cross-sections.

$$\sigma(E) = \frac{\pi}{k^2} P_{\ell=0}(E) \tag{3}$$

   When evaluating cross section data, it is useful to define a quantity that has these gross features taken into account. For neutrons, I have defined what I call an R-factor, and for charged-particle reactions we can define what is called the S-factor. We assume that the R- and S-factors are not strongly energy dependent (i.e. roughly constant).

$$\sigma(E) = R(E) v^{power1} \quad \text{for neutrons} \tag{4}$$

$$\sigma(E) = S(E) \exp(-2\pi\eta) E^{power2} \quad \text{for charged particles} \tag{5}$$

   (e) Find the values of *power*1 and *power*2. I have assumed that the exponent is dominant at low energies for charged-induced reactions.

   Playing a bit, we could define a factor that can be used by both neutrons and charged-particles. Lets call this the JINA-factor.

$$\sigma(E) = J(E) \frac{\pi}{k^2} \frac{2\pi\eta}{\exp(2\pi\eta) - 1} \frac{v}{c} \tag{6}$$

   However, new notations are hard to establish in a field that has been using S-factors and something like R-factors for more than 50 years.

   (f) What are the proportionality factors that would then relate R- and S-factors to JINA-factors?