Homework 2

1. Nuclear Reactions and their Q-values

   Nuclear reactions involve the rearrangement of the constituent neutrons and protons to form new nuclide(s). As in any reactive process there is an energy cost or benefit, that can enhance or inhibit particular reactions (chemists usually speak in terms of an enthalpy of reaction).

   We can calculate the amount of energy released or required for a reaction, by comparing the total mass (i.e. energy) of the initial and final nuclides in the reaction. We will define the Q-value to be the energy released in a reaction, such that it is a positive number for an energetically favored reaction. For simplicity we will consider binary reactions (i.e. reactions between 2 nuclides).

$$\text{reaction}: = A + B \rightarrow C + D \tag{1}$$
$$M_A + M_B = M_C + M_D + Q[A(B,C)D] \tag{2}$$

   Remembering our work from the previous problem, we can substitute the relations for mass excess or binding energy here. Since the total number of neutrons and protons is unchanged in nuclear reactions (charge and mass conservation), we know that the "bulk" mass terms (i.e. $(Z+N)m_u$ and $Z(m_p + m_e) + Nm_n$) will cancel on both sides. We are left with an equation for the Q-value that depends on only the mass excesses or only on the binding energies.

$$\Delta_A + \Delta_B = \Delta_C + \Delta_D + Q[A(B,C)D] \tag{3}$$
$$B_A + B_B = B_C + B_D - Q[A(B,C)D] \tag{4}$$

   A brief side note on some notation. Fusion reactions are those in which several nuclides combine to form a new single nuclide. The energy is released via a photon (γ), which has zero mass. Also, many reactions involve the ^{4}He nucleus (also known as the α-particle), so people's notation may use ^{4}He and/or α. In addition, the "heavy" hydrogens have their own names, the nuclides of ^{2}H and ^{3}H are known as the deuteron (d) and triton (t) (Their neutral atoms known as deuterium and tritium). Few people call the ^{3}He nuclide "helion", but it is present in the literature.

   First, make sure I have written the reactions properly. Make sure that the same number of neutrons and protons are on each side of the reaction. This will verify both charge and mass conservation. I have purposely made 2 mistakes for you to catch. These mistakes are in one of the final nuclides (excluding p, n, α and γ).

   Calculate Q-values for the following reactions. Use both Δ and B methods (for a double check):

   (a) n + p → d + γ
   (b) ^{3}He$(\alpha,\gamma)^7$Be
   (c) ^{3}He(α,p)^{6}Li
   (d) ^{7}Be(p,γ)^{8}B
   (e) ^{15}O$(\alpha,\gamma)^{19}$Ne
   (f) ^{16}O$(\alpha,\gamma)^{20}$Ne
   (g) ^{18}Ne(p,γ)^{19}Na
   (h) ^{18}Ne(p,α)^{15}F
   (i) ^{19}Ne(p,γ)^{20}Na
   (j) ^{19}Ne(p,α)^{17}F

   Generalize the Q-value equation to accommodate more than two nuclides on each side.

   (k) $3\alpha \rightarrow {}^{12}\mathrm{C} + \gamma$
   (l) ^{12}C + ^{16}O → α + ^{24}Mg
   (m) ^{12}C + ^{16}O → p + ^{27}Al
   (n) ^{12}C + ^{16}O → n + ^{27}Si
   (o) ^{12}C + ^{16}O → 2α + ^{20}Na
   (p) ^{12}C + ^{16}O → 2p + ^{26}Mg