Homework 3

1. Cross sections

   Of key importance to experimental nuclear physics is understanding how often (or at what probability) nuclei will react to form new nuclides. To illistrate this imagine a wet ball, moving towards a tissue paper round of radius a, with some speed. It is quite easy to convince yourself that the ball will catch the tissue paper round if and only if it is travelling within the cross-sectional area of the tissue. The bigger the tissue, the more likely the ball is to catch it. The reaction cross section for this reaction can then be expressed numerically as:

   $$\sigma(E) = \pi a^2 \tag{1}$$

   Now, imagine that both the ball and tissue paper can carry an electric charge. Knowing that like-charges repel and unlike-charges attract, we can qualitatively modify this cross section to account for this new interaction. If the ball and tissue have like charges, then they have a force acting on them, trying to push them apart. This can push the ball outside of the tissue papers cross-sectional area before it can react. In effect, it reduces the area of the paper and reduces the chance of hitting the paper. Alternatively, unlike-charges attract each other. This attraction will now pull the ball closer to the paper, thus balls that normally would have zoomed by, now have a greater chance of catching the paper, increasing the cross-section.

   We will adopt this qualitative picture of reactions. Naively we would then assume that nuclear cross-sections similarly scale with the size of the system (R_N, the nuclear radius). However, note that nuclear systems are quantum mechanical and not classical; therefore they are a bit more complicated. One principle idea of quantum mechanics is the idea that particles can behave like waves and vica-versa. The intrinsic wavelength of a particle, is known as the deBroglie wavelength ($\lambda = h/p$ or $\lambdabar = \hbar/p$) and is related to the particle's momentum. For a system of two particles, we use the relative momentum ($\vec{q} = \frac{\mu}{m_1}\vec{p}_1 - \frac{\mu}{m_2}\vec{p}_2$, where the reduced mass is $\mu = \frac{m_1 m_2}{m_1+m_2}$), as opposed to using the total momentum ($\vec{P} = \vec{p}_1 + \vec{p}_2$). One can see that as the energy (and momentum) gets smaller, the deBroglie wavelength gets larger. In fact, this becomes much larger than the actual radii of the nuclides involved. Thus, the fuzziness of quantum mechanics makes the nuclei seem larger than they really are. We can then expect that it is only at higher energies that the size of the nuclides will be important and thus dominate the cross-section. We thus expect the two qualitative cross-section behaviors:

   $$\begin{aligned} \sigma(E) &= \pi\lambdabar^2 \quad \text{when E is small} \qquad (2)\\ \sigma(E) &= \pi R_N^2 \quad \text{when E is large} \qquad (3) \end{aligned}$$

   (a) Find the critical energy, that identifies the transition from the deBroglie regime to the nuclide radius regime, by finding the energy at which these two cross-sections are equivalent. Use $E = q^2/2\mu$. Express this in terms of $\hbar, \mu$ and R_N.

   (b) Now, assume the nuclear radius is $R_N = 1$ fm and the reduced mass is $\mu = m_u$ and find what this energy is in units of MeV. You may want to put the c's back into the masses (e.g. substitute $1 = \frac{c^2}{c^2}$ into your energy formula) and use the constant $\hbar c = 197.326$ MeV fm.

   (c) Nuclear experiments relevant for astrophysics are typically done at energies smaller than 1 MeV. Is it safe to always assume that we are in the deBroglie wavelength regime?

   (d) Assume that $\mu = m_u$ and $R_N = r_0 A^{1/3}$, with $r_0 = 1.4$ fm. Find what value of A, the critical energy is equal to 1 MeV. Round to the nearest integer A.

   (e) Does the deBroglie regime remain valid for small or large A for a cross-section at 1 MeV?